International Engineering Research Journal Page No 372-377

Design and Analysis of Locations and Forces of Cold Bending of Glass Plate

Rajusing D. Rathod [1], Prof. D. G. Joshi [2]

[1] *RMD Sinhgad School of Engineering, Warje, Pune, India.*

[2] *RMD Sinhgad School of Engineering, Warje, Pune.*

***Abstract*— The paper represents the determination of behavior of glass plate under cold bending technique at ambient condition. Analysis of this technique, the bending of a glass plate shows that it won't affect the properties of glass plate after bending. This paper consist of design the technique in which discover appropriate locations on glass plate by applying forces at different points to obtain bending of glass plate up to certain limit for parabolic shape without breakage.**

Parabolic shaped glass bending plate useful for various applications as in solar collectors for heating water. Mostly aluminum plate used in solar collector, it has good efficiency, good reflectivity and better yielding properties than other materials. But its parabolic shape is not perfect. Because of imperfect shape, there is power loss due to no concentration of radiation at focal distance and it deviated.

Now a day's bending of glass plate is done by hot bending but it has a limitation to get parabolic shape and also affect on properties of glass plate

Keywords*— *Cold bending, focal distance, hypermess, hyperview,* linear analysis, *parabolic shape, radioss, reflectivity.

I. Introduction

The solar collector used for small power generation is parabolic solar collectors, as the parabola it has efficient curve for concentrate maximum radiations. This used the aluminum plates bent in parabolic shape. As the aluminum has high reflectivity and good yielding properties than other materials, these are widely used. But still aluminum has not 100% reflectivity. So this drawback can be removed by using material close or equal to 100% reflectivity which is the mirror. Mirror has 100% reflectivity, which can concentrate full radiations which fall on it at particular location. But mirror has poor yielding property which causes the brittle failure of the glass. But still this can sustain yielding up to some extent.

If parabolic shape is not perfect then radiations cannot be able to concentrate at its focal distance and radiations deviates from it. This causes loss of radiations indirectly the loss of power.

In this project we study the behavior of glass under bending at ambient conditions and method of bending it, which forms perfect parabolic profile and used as a solar collector to work it at higher efficiency. This leads the advantages of higher efficiency in generation of power and efficient use of solar energy in less space.

II PLATE

Plates are straight, flat and non-curved surface structure whose thickness is small as compared to their other dimensions. Generally plates are subjected to load condition that causes deflection transverse to the plate. Geometrically, they are bound either by curved or straight lines. A plate has free or simply supported or fixed boundary conditions. The static or dynamic loads carried by plates are mainly perpendicular to the plate surface. The load carrying actions of plates resemble with beams or cables up to a certain extent. So, plates can be approximated by a grid work of beams or by a system of cables, depending on the flexural rigidity of the structure. Plates are of wide use in engineering industry. Nowadays, plates are generally used in varies areas like architectural structures, bridges, hydraulic structures, pavements, containers, airplanes, missiles, ships, instruments and machine parts.

2.1 Plate Equation

Out of several plate theories two are widely accepted and used in the engineering. These theories are

- The Kirchhoff–Love theory of plates (classical plate theory)
- The Mindlin–Reissner theory of plates (shear plate theory)

According to Kirchoff, the assumptions were made by considering a mid-surface plane which helps in representing a three dimensional plate into two dimensional form. According to Kirchhoff, basic assumptions are:

1. Straight lines perpendicular to the flat surface remain straight after deformation.
2. The normal line remains the same length (un stretched).
3. The normal always right angles to the mid surface after deformation.

The plate equation is derived by assume that plate is subjected to lateral forces. Consider following three equilibrium equations

$$\sum M_x = 0 \quad \text{..............2.1}$$
$$\sum M_y = 0 \quad \text{..............2.2}$$
$$\sum P_z = 0 \quad \text{.............2.3}$$

Where M_x and M_y are bending moments and P_z is the external load. The external load P_z which carried by transverse shear forces Q_x , Q_y and bending moments M_x and M_y . The plates generally have noteworthy deviation from the beam and it is due to the presence of twisting moment M_{xy}.

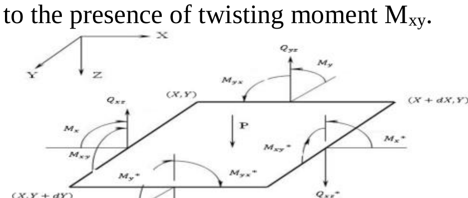


Figure 2.1: Differential Plate with Stress Resultants

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)$$

$$M_y = -D\left(\nu \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

$$M_{xy} = M_{yx} = -D(1-\nu)\left(\nu \frac{\partial^2 w}{\partial x\, \partial y}\right)$$

If the resultant moment at an edge parallel to the X and Y axes is set to zero then the resulting equations after neglecting higher order terms gives equation is

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y} + Q_{xz} = 0$$

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} + Q_{yz} = 0$$

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\, \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -P_z(x, y)$$

$$\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2\, \partial y^2} + \frac{\partial^4 W}{\partial y^4} = \frac{P_z}{D}$$

$$\nabla^4 w = \frac{P}{D}$$

Where,

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2\, \partial y^2} + \frac{\partial^4}{\partial y^4}$$

And

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

Where,

D is flexural rigidity of the plate
E is young's modulus of the plate
h is height of the plate
ν is poisson's ratio

2.2 Boundary conditions

Generally, different types of boundaries are considered for a plate in terms of lateral deflection of the middle surface of the plate, they are: Clamped edge, Simply Supported edge, mixed edge and free edge conditions.

2.3 Deflections of plate as a parabola

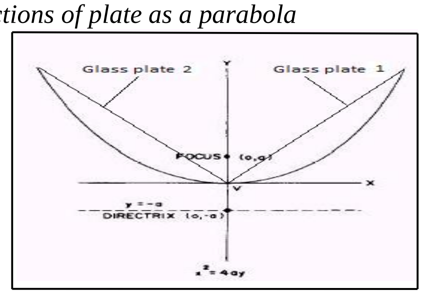


Figure 2.2: The Parabola about y axis

The equation of parabola shown in fig 2 is,

$$x^2 = 4ay$$

In this project we are considering above equation as per fig.2.2. Here two parabolic mirror plate forms a complete parabola of solar collector. Here we are considering only mirrored plate 1 as shown in fig. 2.2. Other side is symmetry. After bending the mirrored plate, combine of these two plates, it will form one parabola. So, main objective is to bend the mirrored plate which is a half of the original parabolic shape.

III DETAILS OF ANALYSIS

Basically there are two types of analysis linear analysis and nonlinear analysis. When deformation in the structure linearly proportional to the load then structure is subjected to the linear static deformation, if not then the structure is consider as subjected to nonlinear static deformation.

The stiffness matrix relating to the load and its response is assumed to be constant for static analysis, but all real world structures behave like nonlinear. The stiffness matrix consists of geometric parameters as length, cross sectional area and a moment of inertia of the section and material properties as elastic modulus etc. The static analysis assume that these parameters do not change when structure is loaded; other hand nonlinear static analysis considered the change in these parameters as load is applied to the structure. These changes are considered into the analysis by rebuilding the stiffness matrix with deformed structure configuration and updated properties after each incremental load application.

We considered three types of nonlinear analysis: Geometric nonlinearities (large displacements), Material nonlinearity and boundary nonlinearities

Following are the different kinds of geometric nonlinearities. If an elements shape changes i.e. area, thickness etc., its individual element stiffness will change is called Large strain. If an elements orientation changes i.e. rotation, the transformation of its local stiffness in to global component will change. Stress stiffening is associated with the tension bending coupling. More the tension in the membrane, more it's bending rigidity or stiffness. If an elements strain produces a significant in-plane stress, the out-of-plane stiffness can be much affected.

All engineering materials are inherently behave as nonlinear as it is not reasonable to characterize a nonlinear material by a single constitutive law for entire range of environment conditions like loading, temperature and rate of deformation. We can simplify the material behaviour to an account for only some effects which are important for the analysis.

Boundary nonlinearity arises when boundary conditions in a FEA model changes during the course of the analysis. Boundary conditions should be added to or removed from the model due to boundary nonlinearity as a analysis progress. This kind of nonlinearity which typically involves contact sets in the model which can get engaged or disengaged as response to applied loads. The load transfer mechanism via contact pairs is complicated phenomenon.

As the bending of the mirrored plate under consideration has large deformation during bending, the analysis of plate falls under the category of stress stiffening nonlinear geometrical analysis.

IV ANALYSIS WORK

The analysis of the mirrored plate is carried out in the Hypermesh11.0 software. This software used for pre-processing and for solving boundary conditions, Radioss software is used. Radioss software gives excellent results for geometrical nonlinear analysis and compatible with Hypermesh. For post-processing Hyperview software used.

4.1 Define the support condition

Analytically if F=1 KN, beam span L= 0.5 m, C/S area A= 10×10 mm^2, E= 2.12×10^{05} N/mm$^{2.}$

By mathematical formula deflection is $\delta = FL^3/48EI$

Thus δ= 14.89 mm

Analysis can be validated by the FEA software Hypermesh 11.0.

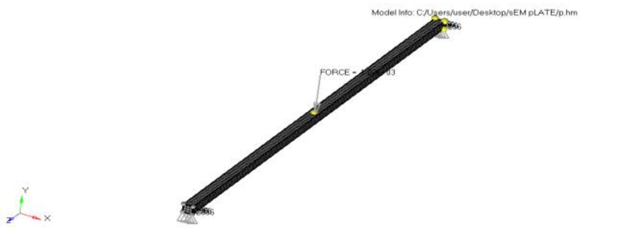


Fig.4.1 a simply supported beam

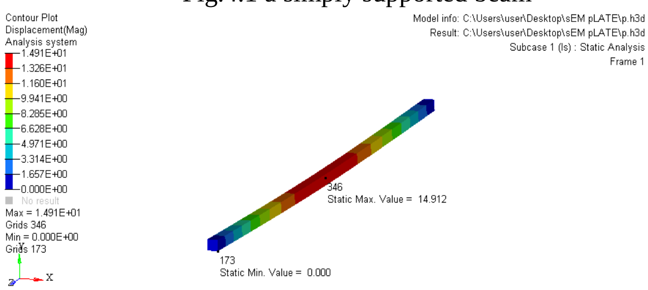


Fig. 4.2 A Static Deflection

4.2 Verify the properties of mirrored glass, validation of software results with practical results

It is necessary to verify properties which relate to the strength of material, as mirror is a brittle material. As we are using the software we must have to check that it's giving the proper results which are very much related with the practical result. To verify this, a small experiment is conducted on a mirrored plate. Specimen specification: 1m × 0.5 m mirror plate, thickness which is 1 mm, E= 70 Gpa, poisons ratio = 0.2, Density = 2.5×10^{-9} Tons/mm ^{3}.

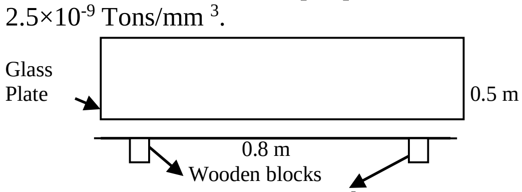


Fig 4.2.1 Experimental setup

A Software Setup:

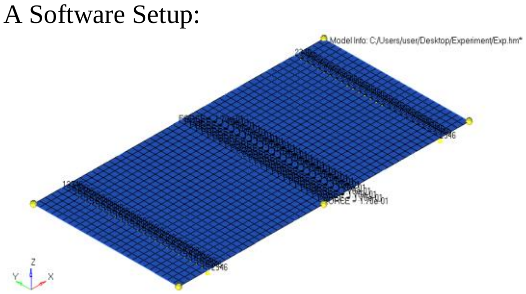
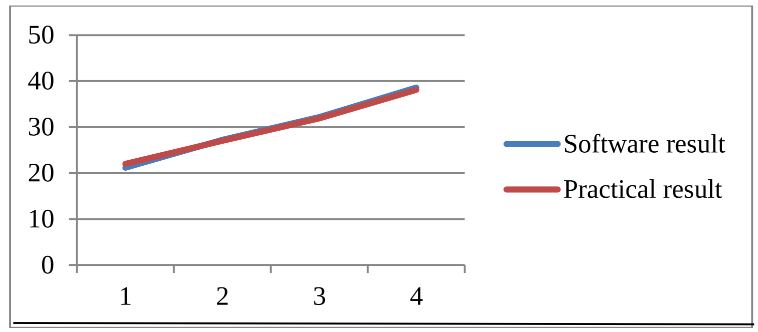


Fig 4.2.2 Software model

Table 4.2.1: Comparison of experimental results with software results

Loading Weight (KN)	Software deflection (mm)	Experimental deflection (mm)	Error (mm)
Self Weight	21.19	22	-0.81
1.65	27.2	27.02	0.18
3.05	32.2	31.9	0.3
4.59	38.6	38.1	0.5



Graph 4.2.1 Comparison of practical and software results

It's find that results are close to the practical values so the software is rightly compatible for this analysis.

4.3 Find the ultimate strength of the mirror plate

To design glass structure the factor of safety while designing must be at least 2. So we know the exact ultimate strength of the mirrored plate. So we conduct small experiment has conducted which is explained below.

To finding out ultimate strength of the glass, practically load is applied in stepwise until glass breaks. Same load is applied on same model and got stress level for that particular load which tend to break the glass.

Specimen: 250 mm × 145 mm mirror plate, Thickness is 1 mm, E= 70 Gpa, poisons ratio = 0.2, Density = 2.5×10^{-9} Tons/mm ^{3}.

Experimental Breaking a load: 22.3 N

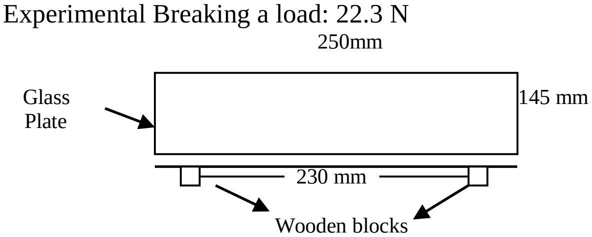


Fig. 4.3.1 Experimental setup

Software Setup

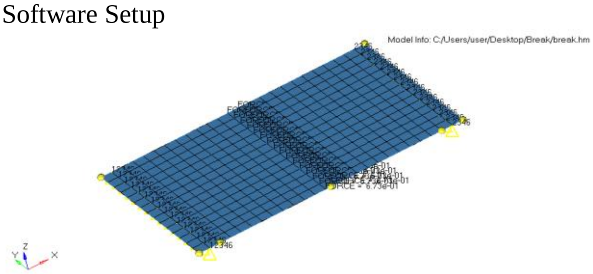


Fig.4.3.2 Software model

The results obtained by software considering 22.3 N as a load:

Deflection

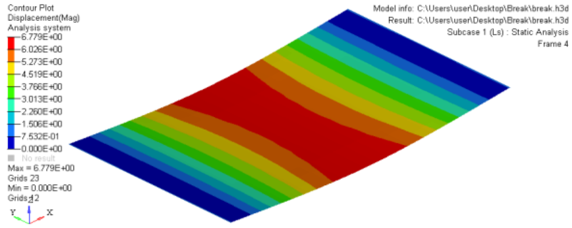


Fig. 4.3.3 Software deflection result

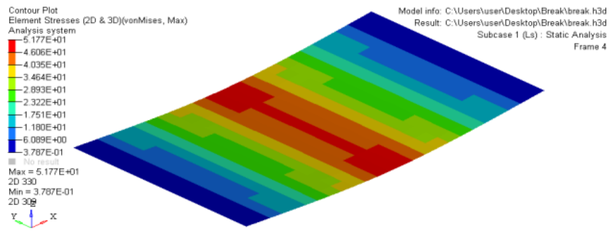


Fig. 4.3.4 Software stress result

The stress value got from the software after applying the 22.3 N loads is 51.78 N/mm2. So we can consider the ultimate strength of the glass is 50 MPa.

So finally properties used for analysis of glass are finalized as:

Modulus of elasticity E = 70 Gpa , Poisson's ratio = 0.2, Density = 2.5×10^{-9} Tons/mm ^{3},Ultimate strength U = 50 MPa.

4.4 Defining Geometrical nonlinearity

The bending of the mirrored plate is categorized as large deflection as compared to thickness of the mirror. Therefore there may be chances of the geometric nonlinearity. The result which obtained from linear analysis may not be confirmed with the practical result due to this geometric nonlinearity. Therefore we must have to check that up to what extend the results may vary due to this kind of nonlinearity. For this small experiments have conducted.

Specimen: 500 mm×10 mm mirror plate, thickness is 1 mm , E=70 Gpa, poisons ratio = 0.2 ,Density = 2.5×10^{-09} Tons/mm^3

A force applied at the end of plate parallel to support.

Fixed support

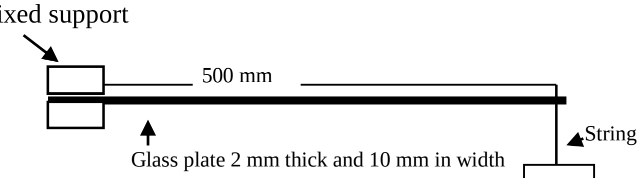


Fig. 4.4.1 Experimental setup

Software Setup :

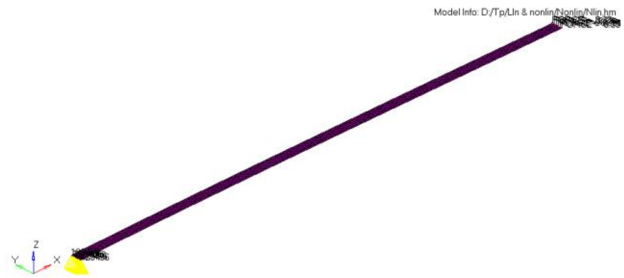
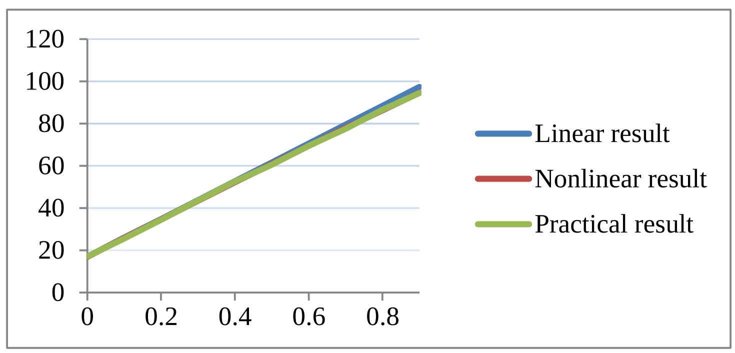


Fig. 4.4.2 Software model

Table 4.4.1 Comparison of experimental and software results

Loading Weight (N)	**Software deflection Linear (mm)**	**Software deflection Geometrical Nonlinear (mm)**	**Experimental deflection (mm)**
Self weight	-	-	-
0.1	16.93	16.92	17.0
0.2	25.88	25.82	25.5
0.3	34.80	34.62	34.5
0.4	43.69	43.43	43.5
0.5	52.62	52.14	52.5
0.6	61.58	60.83	60.5
0.7	70.50	69.43	69.5
0.8	79.40	77.93	77.5
0.9	88.35	86.23	86.5
1	97.27	94.53	94.5



Graph 4.4.1 Comparison of linear, nonlinear and practical values of deformation.

After performing experiment for the support of geometrical nonlinearity it observed that after 60 mm deflection actually effect of the nonlinearity, it differ the linear and nonlinear results above 1 mm and the experimental results which match with the software results.

Thus Geometrical nonlinearity must consider during whole analysis process.

4.5 Decide the focal length of the mirror

The mirrored plate under consideration is 2 mm thickness and 1000 mm span. The ultimate strength of the plate is 50 MPa.

The factor of safety of the mirror should be 2. This assumed on the basis of experience of expertise in the company, that factor of safety 2 is sufficient for mirrored glass plate under static load.

The allowable stress is calculated by formula

$$\sigma\ (\text{allowable}) = \frac{\sigma(\text{ultimate})}{\text{FOS}} = \frac{50}{2} = 25\ \text{MPa}$$

So, the bending of mirrored plate should be up to such a extend that stress should not exceed the limit of 25 MPa stress limit. Tolerance in deflection can be allowed up to +/−2.2 mm.

On the basis of stress limit, we have consider different focal length curvatures and find maximum deflections of mirror required to get the particular profile of the focal length.

Some of the focal length like: for 500 mm focal length maximum deflection required is 95mm, for 1000mm it is 56 mm and for 1500 mm it is 40.2 mm. So analysis is done by considering these deflections and stress levels.

The different stress levels are as shown below:

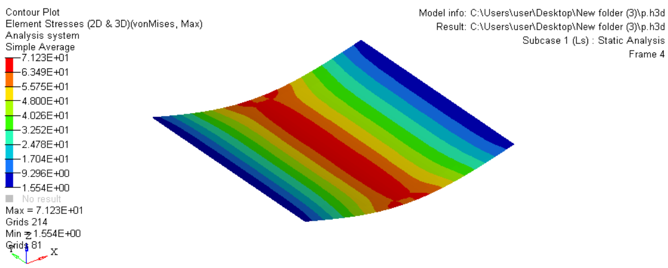


Fig. 4.5.1 Stress level in MPa for 500 mm focal length

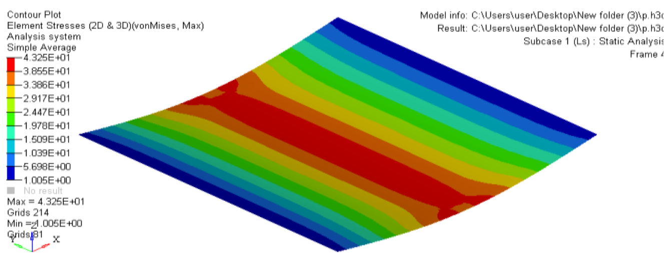


Fig. 4.5.2 Stress level in MPa for 1000 mm focal length:

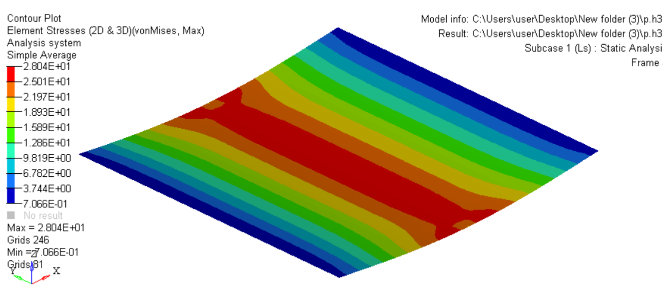


Fig. 4.5.3 Stress level in MPa for 1500 mm focal length

From the above result the maximum stress level in different focal length curves are:

1. For 500 mm focal length, maximum stress is 71.24 MPa.
2. For 1000 mm focal length, maximum stress is43.28 MPa.
3. For 1500 mm focal length, maximum stress is 28.05 MPa.

So, we can consider that bending of mirror is possible for 1500 mm focal length by considering the factor of safety as 2.

4.6 Bending analysis of mirror plate for 1500 mm focal length

For bending the mirrored plate we need to analyze the location and magnitude of forces required to bend it. We need to consider the deflection of the curve from flat position at 9 equidistant points. The deflection required is shown in figure.

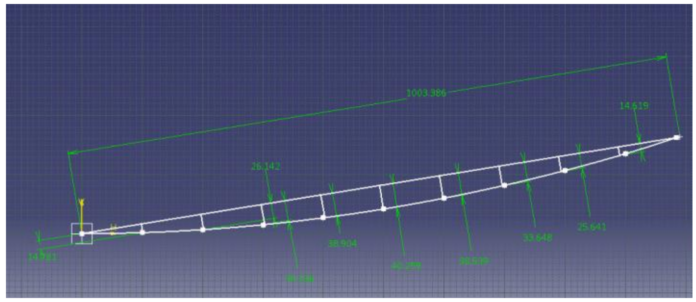
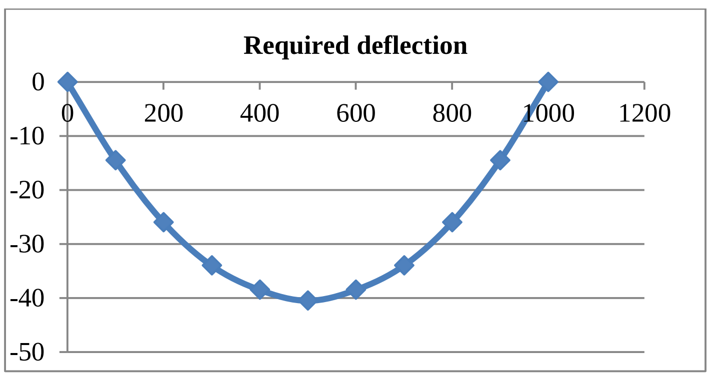


Fig. 4.6.1 Required deflection of plate

Table 4.6.1. Required deflection of plate

Horizontal Distance(mm)	Required Deflection (mm)
0	0
100	14.6
200	25.5
300	34.5
400	38.5
500	40
600	38.5
700	34.5
800	26
900	15
0	0



Graph: 4.6.1 Horizontal distance Vs deflection

Here the deflection is considered as a symmetrical because; the mentioned tolerance is around +/- 2.2 mm. The difference of deflection about symmetry is within 1 mm so it will be consider as a symmetrical structure, even though it is a parabola.

V CONCLUSION

After doing the whole analysis it comes to conclusion that, it is possible to cold bend the mirrored plate for 1500mm focal length. There is nonlinearity available while bending the plates for large deflection and these are accurately simulated with Hypremesh software with using Radioss as a solver. The bending of mirrored plate is carried out easily within the safe stress limit using the mechanism developed during project.

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