Modal Analysis of Pump Rotor System using Finite Element Analysis

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ABSTRACT

In any centrifugal pump rotor system resonance occurs when the operating frequency of the rotating element equals the natural frequency of lateral or transverse vibration. There are chances that the shaft may break leading to serious breakdown of the system. Also excessive shaft deflections may cause large bearing reactions and may lead to bearing failure. Torsional and lateral critical speed analysis plays a major role in determining the natural frequency of the rotor system for safe operation. The main objective of this paper is to calculate critical speeds of the pump rotating system using Finite Element Analysis (FEA) and to verify with respect to different range of operating speeds. The rotordynamic analysis is conducted for a various range of speeds using FEM software (ANSYS 15) taking gyroscopic effect into consideration. FEA software provides Campbell diagram from which critical speeds can be found out. Maintaining a critical speed margin of atleast 20% between the operating speed and the nearest critical speed is a requirement for safe operation. This paper contains the model preparation, static structural analysis in Finite Element Method. Also analytical calculations are performed for the rotor system of a centrifugal pump. Finally, the results obtained from analytical calculations are compared with FEM results and both are verified.

Keywords— Critical speed, Finite Element Analysis, Modal analysis, Rotordynamics

ARTICLE INFO

Article History

Received :18th November 2015
Received in revised form : 19th November 2015
Accepted : 21st November 2015
Published online : 22nd November 2015

I. INTRODUCTION

The critical speed of rotor system is one of the most common phenomenons in rotating machinery which causes excessive vibrations, produce bothering noises, and create unbalance forces on rotors, bearings and couplings. All objects exhibit atleast one natural frequency. It is the frequency at which the object will vibrate if struck. Resonance occurs when the object is repeatedly excited at the natural frequency. It is possible to change the natural frequency of a system by changing any of the factors that affect the size, inertia, or forces in the system. Resonance results in large amplitudes that can be noisy and destructive. It is accepted that good design calls for avoiding such conditions. As a result while designing a machine; modelling and calculations are performed to estimate the natural frequencies of shaft and system. The shaft or rotor is the rotating component of the pump. The shaft is designed in a relatively long geometry to maximize the space available for components such as impellers and seals. Normally pumps are operated at high rotor speeds in order to maximize the power output. Modern high performance pumps usually operate above the first critical speed and this speed is considered to be the most important mode in the system, although it must be avoided in continuous operation at or near the critical speeds. The rotordynamic analysis of the centrifugal pump rotor is performed using Finite Element Analysis (FEA) technology to find the centrifugal pump rotor’s natural frequencies. Machines have to run at higher speeds to meet the higher target of power. The pump shaft will be subjected to higher lateral and torsional vibrations because of gyroscopic effect and centrifugal force of the rotating elements mounted on the shaft. The resonance is
created when the rotor’s natural frequency coincides with the operating frequency. At this point, the deflection of the rotor will be maximum.

Critical speed analysis of the shaft, detecting and diagnosing the resonance condition is required. So it is important to know the reasons of failure of rotor well in advance which will help to reduce the down time. The main objective of this work is to calculate critical speeds of the rotating system using Finite Element Analysis and to verify by experimental verification. The power for the shaft is taken from electric motor. The coupling is used to connect driving shaft of electric motor to drive shaft which is supported between the bearings.

II. LITERATURE REVIEW

Rotordynamics is the study of vibrational behaviour in axially symmetric rotating structures. [6] Devices such as engines, motors, disk drives and turbines develop significant inertia effects that can be analysed to improve the design and decrease the possibility of failure. In a pump rotor, at higher rotational speeds the inertia effects of the rotating parts must be consistently represented in order to accurately predict the rotor behaviour. The history of rotordynamics reviews early development of simple rotor models starting from the Rankine to Jeffcott rotor model and its physical interpretation of different kinds of instabilities in rotor-bearing systems.

![Typical rotor bearing test rig](image)

**Fig. 1 Typical rotor bearing test rig [6]**

Rankine (1869) performed the first analysis of a spinning shaft. He predicted that beyond a certain spin speed the shaft is considerably bent and whirls around in this bent form. He defined this particular speed as the whirling speed of the shaft. In fact, it can be shown that beyond this whirling speed the radial deflection of rankine's model increases without limit, which is not true in actual case. However, Rankine did add the term whirling to the rotor dynamics vocabulary. Whirling may be defined as the movement of the centre of mass of the deflected disc or rotor in a plane perpendicular to the bearing axis. In general, the frequency of whirl, \( n \), depends on the stiffness and damping of the rotor except for the synchronous whirl in which case it is equal to the unbalance excitation force frequency, \( \omega_0 \), i.e., the spin speed of the rotor, and the amplitude as a function of the excitation force’s frequency, \( \omega \), and magnitude. The critical speed, \( c_{\omega} \), occurs when the excitation frequency coincides with the natural frequency, \( n_{\omega} \), and it leads to excessive vibration amplitudes. Rankine neglected the effect of the Coriolis acceleration in his work, which led to improper conclusions that confused engineers for almost five decades.

In order to calculate the critical speeds of cylindrical shafts with several discs and bearings the general theory of Reynolds (Dunkerley, 1895) was applied. The gyroscopic effect was also considered, together with its dependence on speed (i.e., a Campbell diagram). Dunkerley found that as a result of numerous measurements and various calculations, the relationship can be calculated for the first critical speed even for complicated cases.

![Typical Campbell Diagram](image)

**Fig. 2 Typical Campbell Diagram**

III. DESIGN CALCULATIONS

A. Design of shaft:
Shaft even in the absence of external loads will deflect during rotation. At the critical speed the shaft is subjected to violent transverse directional vibrations. The excessive vibrations associated with the critical speed may cause permanent deformation of shaft as well as failure of bearings.

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Maximum torque to transmit, ( T )</td>
<td>14.2 N-m @ 500 rpm</td>
</tr>
<tr>
<td>2</td>
<td>Power rating of motor, ( P )</td>
<td>1 hp = 750 W</td>
</tr>
<tr>
<td>3</td>
<td>Material of construction for shaft</td>
<td>AISI 4140</td>
</tr>
<tr>
<td>4</td>
<td>Allowable shear stress, ( \tau )</td>
<td>45 MPa</td>
</tr>
</tbody>
</table>

To determine: diameter of shaft (\( d \))
By using Torsion equation the diameter of shaft is evaluated.
Thus, \( d = 13.41 \text{ mm} \)
Therefore, minimum diameter selected is 15 mm

B. Bearing reactions:
The bearings along with two masses are placed on to the shaft as shown in the figure 3.

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mass 1, ( M_1 )</td>
<td>2 kg</td>
</tr>
<tr>
<td>2</td>
<td>Mass 2, ( M_2 )</td>
<td>4 kg</td>
</tr>
<tr>
<td>3</td>
<td>Mass of shaft, ( M_s )</td>
<td>1.6 kg</td>
</tr>
</tbody>
</table>
Let the reactions at bearing ends be $R_A$ and $R_B$

![Fig. 3 Supports at end condition](image)

Taking moments @ Bearing A,

$$R_A + R_B = 7.6 \text{ kg}$$

$$M_1 * 350 + M_2 * 450 + M_4 * 500 = R_b * 900$$

Therefore, $R_B = 3.8 \text{ kg}$ and $R_A = 3.8 \text{ kg}$

I. TORSIONAL ANALYSIS OF ROTOR:

Torsional vibrations are important whenever there are large discs on relatively thin shafts. Study of torsional vibrations of rotor is very important where high power transmission and high speed is present. For machine’s having large rotors and flexible shafts (where system natural frequencies of torsional vibrations may be close to or within the source frequency range during operation) torsional vibrations constitute a potential design problem area. In such cases one should ensure the correct prediction of torsional frequencies of machine and frequencies of any torsional load fluctuations should not coincide with torsional natural frequencies.

A. Modelling of rotor system:

![Fig. 4: 3D Model of Rotor system](image)

![Fig. 5: 2D Drawing of Rotor system](image)

Consider a rotor system as shown in figure 4 which consists of a shaft along with two masses supported with two bearings at the end.

Let $I_{p1}$ and $I_{p2}$ be polar mass moment of inertia of the disc mass 1 and 2, respectively.

Torsional stiffness ($k_t$) is given by,

$$k_t = \frac{T}{\theta} = \frac{GJ}{L}$$

Where $G$ = Modulus of rigidity (MPa)

$J$ = Polar moment of Inertia (mm$^4$)

$L$ = length between masses

$J = 1.5708E-08$

$$K_t = \frac{8377.58 \text{ N-m / rad}}{L}$$

Torsional Natural frequency is given by,

$$\omega_n = \sqrt{\frac{(I_{p1} + I_{p2})k_t}{I_{p1}I_{p2}}}$$ ad/sec or $f_n = 248.2 \text{ Hz}$

Hence it is verified that the operating frequency is away from natural frequency.

II. FINE ELEMENT ANALYSIS

The lateral critical speed analysis is only concerned with the rotor and hence the coupling between the pump and motor may be neglected in calculations as the coupling is flexible in nature.

The FE Model is prepared based on following parameters

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Component</th>
<th>ANSYS Element</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Shaft</td>
<td>BEAM188</td>
</tr>
<tr>
<td>2</td>
<td>Mass 1 and 2</td>
<td>MASS21</td>
</tr>
<tr>
<td>3</td>
<td>Bearings</td>
<td>COMBIN14</td>
</tr>
</tbody>
</table>

A. Boundary conditions:

i. All the nodes of the rotor / shaft in the axial direction are constrained.

ii. The nodes at the bearing location have displacement in $Y$ and $Z$ direction and constrained in axial (X direction) as well as torsional direction.

iii. Mass 1 and Mass 2 act as lumped masses on to the shaft at certain distance as shown in figure 5.

iv. The concerned speed range for the analysis is 0 to 3000 rpm.
The FE Model of the rotor system is shown below.

![FE Model of Rotor system](image)

Fig. 6: FE Model of Rotor system

**B. Undamped Natural frequency analysis:**
The stepped shaft with a span of 900 mm is supported on the relatively rigid base through ball bearings. The shaft also carries a balanced disc of 180 and 130 mm diameter having 20 mm thickness at two locations between the two bearings of the shaft. A motor is also connected to the shaft through a flexible coupling to drive the shaft at different speeds. The critical speeds are extracted when rotor is supported only at the bearings by considering the masses of inertias mounted on rotor.

After performing the Modal analysis, the results are plotted below:

![Campbell Diagram](image)

Fig. 7: Campbell Diagram

![Zoomed Campbell Diagram](image)

Fig. 8: Zoomed Campbell Diagram (at 1089 rpm)

![1st Mode shape](image)

Fig. 9: 1st Mode shape (orbit plot)

After performing the harmonic analysis and plotting the graph at mid node of amplitude v/s frequency, we get maximum deformation as desired.

![Frequency response curve at resonance](image)

Fig. 10: Frequency response curve at resonance

**TABLE V**

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Speed (RPM)</th>
<th>Direction</th>
<th>Amplitude (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1089.39</td>
<td>Y</td>
<td>2.72</td>
</tr>
<tr>
<td>2</td>
<td>1091.27</td>
<td>Z</td>
<td>2.74</td>
</tr>
</tbody>
</table>
TABLE VI
MAXIMUM AMPLITUDE AT RESONANCE

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Location</th>
<th>Amplitude (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Shaft</td>
<td>2.74</td>
</tr>
<tr>
<td>2</td>
<td>Bearing</td>
<td>0.04</td>
</tr>
</tbody>
</table>

IV. CONCLUSION

From the above results obtained from the FEA software, it is concluded that the first critical speed for the rotor system is 1091 rpm as obtained from the Campbell diagram. There is no resonance observed near the operating speed range (1450 rpm). The operating speed is 24.8% away from the first critical speed and is acceptable. From the torsional analysis results, it may be concluded that the torsional natural frequency is away from the operating frequency. The maximum amplitude as obtained from the frequency response curve at the bearing locations is 0.04 mm and at shaft location it is 2.74 mm at the resonance condition. The ratio of critical speed to running speed lie entirely outside the unacceptable region as per API 610[8] guidelines hence is safe.

ACKNOWLEDGMENT

I would like to express my deep sense of gratitude towards my guide Prof. A. P. Deshmukh, Dean Academics for his expert guidance, support and encouragement throughout the work.

I would also like to thank Prof. P. A. Makasare, Head of Department of Mechanical Engineering, for constant support and providing me with all possible facilities in the department.

REFERENCES