Post buckling analysis of silos and optimization of additional stiffeners

#1 M.M. Pande, #2 Dr. G.V. Shah

#1 shilpahgujar@gmail.com
#2 nsbiradar123@gmail.com
#3 maheshwarchammalli@gmail.com

#1 Department of mechanical engineering, Savitribai Phule Pune University, Pune, Dhole Patil College of Engineering, Wagholi, Pune, India.

ABSTRACT

Silos are probably the common form of large engineering shell structure in service. The high rate of structural failure in these structures must be considered by the designer and the complexity of their behavior. The paper presents buckling and post-buckling of silos under axial compression using a time-integrated finite element model as well as the analysis and Experimental results. Since stiffeners have the vital role to support the pressure vessel and to maintain its stability, they should be designed for load and internal pressure of the vessel. A model of vertical pressure vessel and stiffeners are created and stresses are evaluated using mathematical approach and ANSYS software. The analysis reveals the zone of high localized stress at the junction of the pressure vessel and stiffeners due to operating conditions. The results obtained by both the methods are compared. The analyses provide significant new insight into the mechanisms underpinning collapse behavior of the shells. The tests which are carried out on cylindrical silo models, the measurement of the initial imperfections and the numerical analysis of the shells are presented.

Keywords— ANSYS, Axial pressure, post-buckling, Silos, Stiffeners.

I. INTRODUCTION

Large steel silos are typical kind of thin walled structures which are widely used for storing huge quantities of granular solids in industry and agriculture. Silos are widely used in agricultural, mining and manufacturing industries to store grain (maize and soya beans), fluids (oil and fuel), cement and platinum ores. The sizes of engineered silos may vary from capacities less than 10 tonnes to the largest containing as much as 1,00,000 tonnes. These steel silos are usually with large diameter to thickness ratios, which is particularly vulnerable to buckling due to internal pressure, wind pressure. The critical load capacity of cylindrical shells, subjected to uniform pressure, depends on two geometric slender ratios of “length to radius” (L/R) and “Radius to thickness” (R/T). Therefore, the geometric imperfections would influence the buckling and post-buckling behaviour of these structures. The experimental studies show that the buckling strength of ideal shells without geometric imperfections is much more than that of imperfect shells. Therefore the imperfection sensitivity of these structures must be considered carefully and properly.

1.2 Objective of project:

Objective of the project is to study the buckling and post-buckling behaviour of silo and verify its safety against buckling failure to validate the design. Additionally, to check whether addition of stiffeners may improve the buckling characteristics of the silo is also the aim of the thesis. Thus following are objectives of our project:

• To do linear buckling analysis of silo
• To do nonlinear buckling analysis of silo
• To experiment with stiffeners (sizes and location) to check for improvement.

II. LITERATURE SURVEY

The stability of conical shells under axial compression has been studied in the past both theoretically
and experimentally by several investigators. The first shell buckling problem solved was cylindrical shells under axial compression.

S Aghajari (1) has studied the buckling and post-buckling behaviour of thin walled cylindrical steel shells with varying thickness subjected to uniform external pressure. He found that the buckling capacity of the thin walled cylindrical steel shells, with thickness with variation in height will increase.

Chawalit Thinvongpituk (2), have studied the buckling of axially compressed conical shells of linearly variable thickness using structural model. He concluded that the reduction of the buckling resistance of cylindrical and conical shells due to their small variable thickness is proportional to the thickness reduction parameter and can be expressed in a simple linear equation.

A Spagnoli (3), have studied the elastic buckling and post buckling behaviour of widely stiffened conical shells under axial compression. Critical load increases as the panel width decreases in line with the behaviour of the cylindrical panel.

M Shariati (4), have studied the experimental results on buckling and post buckling behaviour of cylindrical panels with clamped and simply supported ends and come to the conclusion that by increasing the length of the panels, the buckling load decreases slightly.

III. BUCKLING & POST BUCKLING

3.1 Buckling

Buckling is that mode of failure when the structure experiences sudden failure when subjected to compressive stress. When a slender structure is loaded in compression, for small loads it deforms with hardly any noticeable change in the geometry and load carrying capacity. At the point of critical load value, the structure suddenly experiences a large deformation and may lose its ability to carry load. This stage is the buckling stage. The structural instability of buckling can be categorized as:

1. Bifurcation buckling
2. Limit load buckling

In Bifurcation buckling, the deflection when subjected to compressive load, changes from one direction to a different one. The load at which buckling occurs is the Critical Buckling Load. The deflection path that occurs prior to the bifurcation is called as the Primary Path and that after bifurcation is called as secondary or post buckling path. In Limit Load Buckling, the structure attains a maximum load without any previous bifurcation, i.e. with only a single mode of detection.

3.2 Post Buckling

Post buckling stage is a continuation of the buckling stage. After the load reaches its critical value the load value may not change or it may start decreasing, while deformation continues to increase. In some cases the structure continues to take more loads after certain amount of deformation, to continue increasing deformation which eventually results in a second buckling cycle. Post buckling analysis being non-linear, we obtain far more information than we obtain from linear Eigen-value analysis.

The non-linear load-displacement relationship, which can be a result of the stress strain relationship with a nonlinear function of stress, strain and time. The changes in geometry due to large displacements; irreversible structural behaviour upon removal of external loads; change in the boundary conditions such as change in the contact area and the influence of loading sequence on the behaviour of the structure, requires a nonlinear structural analysis. The nonlinear structural analysis depends upon the various structural nonlinearities. The structural nonlinearities can be classified as, a geometric nonlinearity, a material nonlinearity and a contact or a boundary nonlinearity.

IV. PLATE BENDING THEORY

Expression for stresses induced in rectangular plate subjected to Uniformly Distributed Load.

Let a rectangular plate subjected to UDL and ‘z’ be the half of the thickness of plate.

Let ‘w’ be the deflection of plate, ‘u’ and ‘v’ be the displacements along ‘x’ and ‘y’ axes.

Hence, the displacement along ‘x’ direction,

\[ u = -z \frac{\partial^2 w}{\partial x^2} \]  

Similarly, the displacement along ‘y’ direction,

\[ v = -z \frac{\partial^2 w}{\partial y^2} \]  

As per strain displacement relations, strains are given by-

\[ \varepsilon_x = \frac{\partial u}{\partial x} \]
\[ \varepsilon_y = \frac{\partial v}{\partial y} \]
\[ \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \]  

Put (1) and (2) in equation (3),

\[ \varepsilon_x = -z \frac{\partial^2 w}{\partial x^2} \]
\[ \varepsilon_y = -z \frac{\partial^2 w}{\partial y^2} \]
\[ \gamma_{xy} = -z \frac{\partial^2 w}{\partial x \partial y} - z \frac{\partial^2 w}{\partial x \partial y} \]  

Figure 4.1. Plate definition

Figure 4.2. Deflection of plate in the x-z direction.
From plane stress condition,
\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = \begin{bmatrix}
E/1-\nu^2 & 1 & 0 \\
1 & E/1-\nu^2 & 0 \\
0 & 0 & 1-\nu/2
\end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} \tag{5}
\]
\[
\sigma_x = \frac{E}{1-\nu^2} (\varepsilon_x + \nu \varepsilon_y) \\
\sigma_y = \frac{E}{1-\nu^2} (\varepsilon_y + \nu \varepsilon_x) \\
\tau_{xy} = -\frac{E}{1-\nu/2} (1-\nu) \frac{\partial^2 \omega}{\partial x \partial y}
\]
Substituting equation (5) in equation (6)
\[
\sigma_x = \frac{-Ez}{1-\nu^2} \left( \frac{\partial^2 \omega}{\partial x^2} - \nu \frac{\partial^2 \omega}{\partial y^2} \right) \tag{7a}
\]
\[
\sigma_y = \frac{-Ez}{1-\nu^2} \left( \frac{\partial^2 \omega}{\partial y^2} + \nu \frac{\partial^2 \omega}{\partial x^2} \right) \tag{7b}
\]
\[
\tau_{xy} = \frac{-Ez}{1-\nu^2} (1-\nu) \frac{\partial^2 \omega}{\partial x \partial y}
\]
Figure 4.3. Variation in moments and forces

Let, ‘M’ be bending moment and ‘M_{xy}’ be shear force.
According to beam theory M_x, M_y, M_{xy} are given by the following relations:
\[
M_x = \int_{-t/2}^{t/2} \sigma_x z \, dz \tag{8}
\]
\[
M_y = \int_{-t/2}^{t/2} \sigma_y z \, dz \tag{9}
\]
\[
M_{xy} = \int_{-t/2}^{t/2} \tau_{xy} z \, dz \tag{10}
\]
Substituting equation (7a), (7b) and (7c) in equation (8), (9) and (10)
\[
\Rightarrow M_x = \int_{-t/2}^{t/2} \frac{-Ez}{1-\nu^2} \left( \frac{\partial^2 \omega}{\partial x^2} - \nu \frac{\partial^2 \omega}{\partial y^2} \right) \, dz
\]
\[
\therefore M_x = \frac{-Ez}{1-\nu^2} \left( \frac{\partial^2 \omega}{\partial x^2} + \nu \frac{\partial^2 \omega}{\partial y^2} \right) \left( \frac{t^3}{12} \right) \tag{11}
\]
Let,
\[
D = \frac{Et^3}{12(1-\nu^2)}
\]
Where, D=Flexural rigidity of plate.
\[
M_x = D \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial \omega}{\partial y} \right) \tag{12a}
\]
Similarly, we have
\[
M_y = D \left( \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial \omega}{\partial x} \right) \tag{12b}
\]
\[
M_{xy} = D(1-\nu) \frac{\partial^2 \omega}{\partial x \partial y} \tag{12c}
\]
Comparing (7a) and (12a)
\[
\left( \frac{\partial^2 \omega}{\partial x^2} + \nu \frac{\partial^2 \omega}{\partial y^2} \right) = \frac{\sigma_x}{Ez} = -\frac{12 M_x}{Et^3} \left( 1-\nu^2 \right) \tag{13}
\]
Comparing (7b) and (12b)
\[
\frac{\partial^2 \omega}{\partial y^2} - \nu \frac{\partial^2 \omega}{\partial x^2} = \frac{\sigma_y}{Ez} = -\frac{12 M_y}{Et^3} \left( 1-\nu^2 \right) \tag{14}
\]
Comparing (7c) and (12c)
\[
\frac{\partial^2 \omega}{\partial x \partial y} = \frac{\tau_{xy}}{Ez} = -\frac{\sigma_{xy}}{Ez} = \frac{M_{xy}}{Et^3} \left( 1-\nu^2 \right) \tag{15}
\]
In simple beam theory, moments are easily determined from equilibrium. However, for plate this is not easy method. Consider the \textit{tdxdy} element. Whereupon the lateral load
\[
P_{x,y} = \sum \text{F}_x = \sum \left( \frac{\partial V_x}{\partial x} \right) dy + \sum \left( \frac{\partial V_y}{\partial y} \right) dx + p(x,y) \text{dxdy} = 0 \tag{16}
\]
Dividing by dxdy gives-
\[
\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + p(x,y) = 0 \tag{19}
\]
Summing moments about an axis parallel to the axis through the element center gives:

\[ \sum M_z = \left( \frac{\partial M_{xy}}{\partial x} \right) dy - \left( \frac{\partial M_{zy}}{\partial y} \right) dx + (V_y dx) dy = 0 \]  

Or

\[ V_y = \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{zy}}{\partial y} \]  

(22)

Substituting an equation (21) and (22) in equation (20),

\[ \frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -P_{x,y} \]  

(23)

Finally substituting equation (12) into (23) and simplifying results in

\[ \frac{\partial^4 \omega}{\partial x^4} + 2 \frac{\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} = \frac{P_{x,y}}{D} \]

Where, \( P_{x,y} \) = Pressure applied on the surface of the plate in \( x - y \) plane.

Tabulated solutions of Uniformly Loaded rectangular plates-
1. The terms ‘a’ and ‘b’ are the dimensions of the sides of the plate.
2. The load \( P \) is a uniform pressure applied along the entire plate.
3. The maximum stress and deflection are given by the formulas.

\[ \sigma_{max} = K_1 P_0 \left( \frac{b}{t} \right)^2 \]  

(24)

\[ \omega_{max} = K_2 \frac{P_0 b^4}{E t^3} \]  

(25)

Where, \( K_1 \) and \( K_2 \) are numerical constants values.

4.1 Mathematical modelling
A water tank 3.60m deep and 2.70m square is to be made of structural steel plate. The sides of the tank are divided into nine panels by two vertical supports (or stiffeners) and two horizontal supports that are, each panel is 0.90m wide and 1.20m high, thickness 6.79mm and average head of water on a lower panel 3.00m figure 1. Calculate the maximum deflection of the panel.

\[ \omega_{max} = 5.79 \text{mm} \]

\[ \therefore \omega_{max} = 5.79 \text{mm} \]

\[ \text{Anslysis Result=} \omega_{max}6.3869 \text{mm} \]

### ANSYS SOLUTION
For low pressure applications (P < 1.4 mpa)

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Input Parameters</th>
<th>Expression</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( V_p = 500 \text{ m}^3 )</td>
<td>( V_p = 0.95 \times NTD )</td>
<td>( \text{Nil} = 12,926 \text{ m} )</td>
</tr>
<tr>
<td>2</td>
<td>( V_s = 20 \text{ m}^3, V_b = 5 \text{ m}^3, R = 3.6 \text{ m} )</td>
<td>( V_s + V_b = (0.86 \times L_b) \times 0.14 \text{ m} )</td>
<td>( t = 9.47 \text{ mm} )</td>
</tr>
<tr>
<td>3</td>
<td>( P = 0.2478 \text{ mpa}, R = 3.6 \text{ m}, S = 137.895, E = 1, ca = 3 )</td>
<td>( t = \frac{P \times R}{S - 0.6P + ca} )</td>
<td>( t = 6 \text{ mm} )</td>
</tr>
<tr>
<td>4</td>
<td>( S = 137.895, E = 1, P = 0.2478 \text{ mpa}, R_n = 150 \text{ mm}, nca = 3 \text{ mm} )</td>
<td>( T_{reqN} = \frac{P_n R_n}{S - 0.6P + nca} )</td>
<td>( t = 32698 \text{ mm} )</td>
</tr>
<tr>
<td>5</td>
<td>( d = 0.3 \text{ m}, T_{req} = 9.476 \text{ mm}, t_n = 3 \text{ mm}, F_{t1} = 1 )</td>
<td>( A_t = 1 \times d \times T_{req} = 2.864 \times 3 \text{ mm} \times 9.476 )</td>
<td>( t = (1 - \frac{C}{E}) \times \pi \times P )</td>
</tr>
<tr>
<td>6</td>
<td>( n = 12, F_{t1} = 1, d = 0.3 \text{ m}, t_n = 3 \text{ mm}, F_{t1} = 1, E = 1 )</td>
<td>( A_1 = \max (d, 2 \times A_{0.75} + 2.57 \times 0.25 \times B_r \times r_t) )</td>
<td>( h = 14.288 \text{ m} )</td>
</tr>
<tr>
<td>7</td>
<td>( F_{t2} = 1, T_{reqN} = 4.5 \text{ mm}, t_n = 3 \text{ mm}, t_{mn} = 1.5 \text{ mm}, L_p = 450 \text{ mm} )</td>
<td>( A_1 = \min [t_n, t_{mn}] \times \pi \times L_p \times \frac{P_n R_n}{S - 0.6P + nca} )</td>
<td>( T_{req} = 4.1626 \text{ n} )</td>
</tr>
<tr>
<td>8</td>
<td>( n = 12, t_n = 3 \text{ mm}, t_0 = 0, F_{t2} = 1, E = 1 )</td>
<td>( A_3 = \min [5 \times A_0, t_0 F_{t2}] \times (5 \times t_0) \times \frac{P_n R_n}{S - 0.6P + nca} )</td>
<td>( 2842.8 )</td>
</tr>
<tr>
<td>9</td>
<td>( A_1 = 757.2 \text{ mm}^2, A_2 = 22.5 \text{ mm}^2, A_3 = 0, A_4 = 0 )</td>
<td>( A_a = A_1 + A_2 + A_3 - 779.7 \text{ mm}^2 )</td>
<td>( 2842.8 )</td>
</tr>
<tr>
<td>10</td>
<td>( D_p = 1292.6, R_p = 646.3 \text{ mm} )</td>
<td>( T_{reqP} = \frac{P_n R_p}{S - 0.6P + ca} )</td>
<td>( T_{reqP} = 150 \text{ mm} )</td>
</tr>
<tr>
<td>4</td>
<td>( t_n = 3 \text{ mm}, D_p = 1292.6, d = 0.3 \text{ m}, t_e = 6, F_{t2} = 1 )</td>
<td>( A_3 = (D_p - d) - 2A_3 = -91.18 \text{ mm} )</td>
<td>C = 0.33 m</td>
</tr>
<tr>
<td>5</td>
<td>( A_1 = 757.2 \text{ mm}^2, A_2 = 22.5 \text{ mm}^2, A_3 = 0, A_4 = 55919.6 \text{ mm}^2 )</td>
<td>( A_3 = A_1 + A_2 + A_3, A_3 = 779.7 \text{ mm}^2 )</td>
<td>C min = 0.20</td>
</tr>
</tbody>
</table>

**DESIGN OF FLAT HEAD:**

**FINAL DIMENSIONS OF SILO:**

Thickness of shell = 12 mm
Diameter of shell = 7200 mm
Thickness of nozzle = 6 mm
Diameter of nozzle = 300 mm
Thickness of reinforcing pad = 6 mm
Diameter of reinforcing pad = 648 mm
Thickness of flat head = 134.72 mm  
Internal radius of flat head = 10 mm

V. Conclusion

The theoretical & ANSYS values are very close to each other. The results obtained after linear and nonlinear analysis, is calculated as critical buckling capacity of silo equal to 16 MPa, which is very higher value than actual internal pressure value 0.2498 MPa for which silo is designed. Hence, silo is safe against buckling failure.

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References


